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# Application of Differential Equations using First Order

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**Abstract**

Mathematical concepts and various techniques are presented in a clear, logical, and concise manner. Various visual features are used to highlight focus areas. Complete illustrative diagrams are used to facilitate mathematical modeling of application problems. Readers are motivated by a focus on the relevance of differential equations through their applications in various engineering disciplines. Studies of various types of differential equations are determined by engineering applications. Theory and techniques for solving differential equations are then applied to solve practical engineering problems. Detailed step-by-step analysis is presented to model the engineering problems using differential equations from physical principles and to solve the differential equations using the easiest possible method. Such a detailed, step-by-step approach, especially when applied to practical engineering problems, helps the readers to develop problem-solving skills.

**Keywords**

Partial, Linear and Non Linear Order

## I. Introduction

A differential equation is a mathematical equation that relates some functionwith its derivatives. In applications, the functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two. Because such relations are extremely common, differential equations play a prominent role in many disciplines including engineering, physics, economics, and biology.

In pure mathematics, differential equations are studied from several different perspectives, mostly concerned with their solutions—the set of functions that satisfy the equation. Only the simplest differential equations are solvable by explicit formulas; however, some properties of solutions of a given differential equation may be determined without finding their exact form.

## II. Types

Differential equations can be divided into several types. Apart from describing the properties of the equation itself, these classes of differential equations can help inform the choice of approach to a solution. Commonly used distinctions include whether the equation is: Ordinary/Partial, Linear/Non-linear, and Homogeneous/Inhomogeneous. This list is far from exhaustive; there are many other properties and subclasses of differential equations which can be very useful in specific contexts.

### A. Ordinary Differential Equations

An Ordinary Difference Equation (ODE) is an equation containing a function of one Independent variable and its derivatives. The term "ordinary" is used in contrast with the term partial different equation which may be with respect to more than one independent variable.

Linear differential equations, which have solutions that can be added and multiplied by coefficients, are well-defined and understood, and exact closed-form solutions are obtained. By contrast, ODEs that lack additive solutions are nonlinear, and solving them is far more intricate, as one can rarely represent them by elementary functions in closed form: Instead, exact and analytic solutions of ODEs are in series or integral form. Graphical and Numerical method, applied by hand or by computer, may approximate solutions of ODEs and perhaps yield useful information, often sufficing in the absence of exact, analytic solutions.

### B. Partial Differential Equations

A Partial differential Equation (PDE) is a differential equation that contains unknown multi variable functions their partial devices (This is in contrast to ordinary, which deal with functions of a single variable and their derivatives.) PDEs are used to formulate problems involving functions of several variables, and are either solved in closed form, or used to create a relevant computer model PDEs can be used to describe a wide variety of phenomena such as sound, heat, electronics, fluid and elsticity , or quantum. These seemingly distinct physical phenomena can be formalized similarly in terms of PDEs. Just as ordinary differential equations often model one-dimensional dynamically systems, partial differential equations often model multi dementia systems.

### C. Linear Differential Equations

A Linear differential equations if the unknown function and its derivatives with its degree 1 (products of the unknown function and its derivatives are not allowed) and non leaner otherwise. The characteristic property of linear equations is that their solutions form an affine subsence of an appropriate function space, which results in much more developed theory of linear differential equations.

Homogeneous Linear differential equations are a subclass of linear differential equations for which the space of solutions is a linear subspace i.e. the sum of any set of solutions or multiples of solutions is also a solution. The coefficients of the unknown function and its derivatives in a linear differential equation are allowed to be (known) functions of the independent variable or variables; if these coefficients are constants then one speaks of a constant coefficient linear differential equation.

### D. Non-linear Differential Equations

Non-linear differential equations are formed by the products of the unknown function and its derivatives are allowed and its degree is > 1. There are very few methods of solving nonlinear differential equations exactly; those that are known typically depend on the equation having particular symmetries. Nonlinear differential equations can exhibit very complicated behavior over extended time intervals, characteristic of chaos. Even the fundamental questions of existence, uniqueness, and extendibility of solutions for nonlinear differential equations, and well-posednes Non-linear differential equations [edit]

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Linear differential equations frequently appear as approximations to nonlinear equations. These approximations are only valid under restricted conditions. For example, the harmonic oscillator equation is an approximation to the nonlinear pendulum equation that is valid for small amplitude oscillations (see below).

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The trigonometric functions

The Pythagorean trigonometric identity is

sin2 $x$ + cos2 $x$ = 1,

and the addition theorems are

sin($x$ + $y$) = sin($x$) cos($y$) + cos($x$) sin($y$),

cos($x$ + $y$) = cos($x$) cos($y$) − sin($x$) sin($y$).

Also, the values of sin $x$ in the first quadrant can be remembered by the rule of quarters, with 0° = 0, 30° = $\pi$/6, 45° = $\pi$/4, 60° = $\pi$/3, 90° = $\pi$/2:

$$\sin 0^\circ = \sqrt{\frac{0}{4}}, \qquad \sin 30^\circ = \sqrt{\frac{1}{4}}, \qquad \sin 45^\circ = \sqrt{\frac{2}{4}},$$

$$\sin 60^\circ = \sqrt{\frac{3}{4}}, \qquad \sin 90^\circ = \sqrt{\frac{4}{4}}.$$

The following symmetry properties are also useful:

sin($\pi$/2 − $x$) = cos $x$, cos($\pi$/2 − $x$) = sin $x$;

and sin(−$x$) = −sin($x$), cos(−$x$) = cos($x$).

## 1. Modelling with First-order Equations

Applying Newton's law of cooling In Block 19.1 we introduced Newton's law of cooling. The model equation was

dθ dt = −k(θ − θs)

where θ = θ(t) is the temperature of the cooling object at time t, θs the temperature of the environment (assumed constant) and k is a thermal constant related to the object. Let θ0 be the initial temperature of the liquid, i.e. θ = θ0 at t = 0.

Try each part of this exercise Solve this initial value problem. Part (a) Separate the variables to obtain an equation connecting two integrals A

Part (a) Separate the variables to obtain an equation connecting two integrals

Part (a) Nowintegrate both sides of this equation

Part (a) Apply the initial condition and take exponentials to obtain a formula for θ

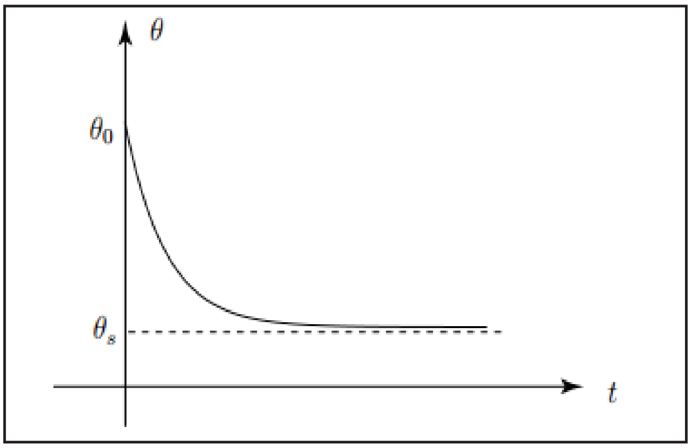
Hence $\ln(\theta - \theta_s) = -kt + \ln(\theta_0 - \theta_s)$ so that $\ln(\theta - \theta_s) - \ln(\theta_0 - \theta_0) = -kt$

Thus, rearranging and inverting, we find:

$$\ln\left(\frac{\theta - \theta_s}{\theta_0 - \theta_s}\right) = -kt \qquad \therefore \qquad \frac{\theta - \theta_s}{\theta_0 - \theta_s} = e^{-kt}$$

and so, finally, $\boldsymbol{\theta} = \theta_s + (\theta_0 - \theta_s)e^{-kt}$.

The graph of $\theta$ against $t$ is shown in the figure below.




We see that as time increases (t → ∞), then the temperature of the object cools down to that of the environment, that is: θ → θs. Note that we could have solved (1) by the integrating factor method.

Part (a) Write the equation as

dθ dt + k θ = kθs

What is the integrating factor for this equation? Answer Multiplying (2) by this factor we find that

ekt dθ dt + kektθ = kθsekt or, rearranging, d dt (ektθ) = kθsekt

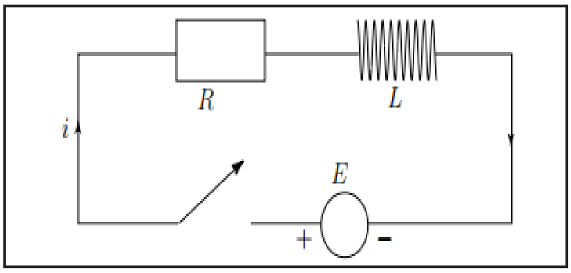
Part (b) Nowintegrate this equation and apply the initial condition

Hence θ = θs + C e−kt. Then, applying the initial condition: when t = 0, θ = θs + C so that C = θ0 − θs and, finally, θ = θs + (θ0 − θs)e−kt,

Electrical circuits Another application of first-order differential equations arises in the modelling of electrical circuits the differential equation for the RL circuit of the figure belowwas shown to be

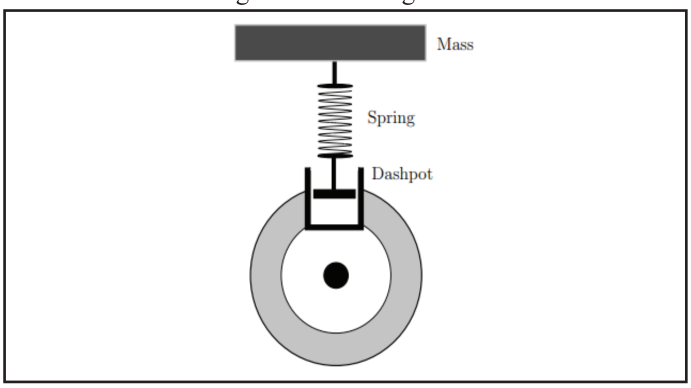
$$L\frac{\mathrm{d}i}{\mathrm{d}t} + Ri = E$$

L di dt + Ri = E in which the initial condition is i = 0 at t = 0.




## III. Modelling Free Mechanical Oscillations

Consider the following schematic diagram of a shock absorber:




The equation of motion can be described in terms of the vertical displacement x of the mass. Let m be the mass, k dx dt the

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damping force resulting from the dashpot and nx the restoring force resulting from the spring. Here, k and n are constants. Then the equation of motion is

$$m\frac{d^2x}{dt^2} = -k\frac{dx}{dt} - nx.$$

Suppose that the mass is displaced a distance x0 initially and released from rest. Then at t = 0, x = x0 and dx dt = 0. Writing the differential equation in standard form:

$$m\frac{d^2x}{dt^2} + k\frac{dx}{dt} + nx = 0.$$

We shall see that the nature of the oscillations described by this differential equation, depends crucially upon the relative values of the mechanical constants m, k and n.

### Now do this exercise

Find the auxiliary equation of this differential equation and solve it. The value of k controls the amount of damping in the system. We explore the solution for two particular values of k.

No damping

If k = 0 then there is no damping. We expect, in this case, that once motion has started it will continue for ever. The motion that ensues is called simple harmonic motion. In this case we have

$$\lambda = \frac{\pm\sqrt{-4mn}}{2m},$$

that is,

$$\lambda = \pm\sqrt{\frac{n}{m}}\,\mathrm{i} \qquad \text{where} \quad \mathrm{i} = \sqrt{-1}.$$

In this case the solution for the displacement $x$ is:

$$x = A\cos\left(\sqrt{\frac{n}{m}}\,t\right) + B\sin\left(\sqrt{\frac{n}{m}}\,t\right)$$

where $A, B$ are arbitrary constants.

Now do this exercise

Nowapply the initial conditions to find the unique solution:

Light damping

If $k^2 - 4mn < 0$, i.e. $k^2 < 4mn$ then the roots of the auxiliary equation are complex:

$$\lambda_1 = \frac{-k + i\sqrt{4mn - k^2}}{2m} \qquad \lambda_2 = \frac{-k - i\sqrt{4mn - k^2}}{2m}$$

Then, after some rearrangement:

$$x = e^{-kt/2m}\left[A\cos pt + B\sin pt\right]$$

in which $p = \sqrt{4mn - k^2}/2m$.

## III. Modelling Forced Mechanical Oscillations

uppose nowthat the mass is subject to a force f(t) after the initial disturbance. Then the equation of motion is

$$m\frac{d^2x}{dt^2} + k\,\frac{dx}{dt} + nx = f(t)$$

Consider the case f(t) = F cos ωt, that is, an oscillatory force of magnitude F and angular frequency ω. Choosing specific values for the constants in the model: m = n = 1, k = 0, and ω = 2 we find

$$\frac{d^2x}{dt^2} + x = F\cos 2t$$

part of this exercise

Part (a) Find the complementary function for this equation.

Part (b) Nowfind a particular integral for the differential equation.

Part (c) Finally, apply the initial conditions to find the solution for the displacement x. Answer If the angular frequency ω of the applied force is nearly equal to that of the free oscillation the phenomenon of beats occurs. If the angular frequencies are equal we get the phenomenon of resonance. Both these phenomena are dealt with in the Computer Exercises. Note that we can eliminate resonance by introducing damping into the system. See the Computer Exercises.

## IV. Computer Exercise or Activity

For this exercise it will be necessary for you to access the computer package DERIVE. To solve a differential equation using DERIVE it is necessary to load what is called a Utility File. In this case you will either need to load ode1 or ode2. To do this is simple. Proceed as follows: In DERIVE, choose File:Load:Math and select the file (double click) on the ode1 or ode2 icon. This will load a number of commands which enable you to solve first-order and second-order differential equations. You can use the Help facility to learn more about these if you wish.

Also note that many of the differential equations presented in this Block are linear differential equations having the general form

$$\frac{dy}{dx} + p(x)y = q(x) \qquad y(x0) = y0$$

Such equations can also be solved in DERIVE using the command Linear1(p,q,x,y,x0,y0) or by using the related command Linear1 Gen(p,q,x,y,c) for a solution to a differential equation without initial conditions but which contains a single arbitrary constant c. Also use the Help command to find out about the more general commands Dsolve1(p, q, x, y, x0, y0) and Dsolve1 Gen(p, q, x, y, c) used for solving very general first-order ordinary differential equations. For second-order equations use the DERIVE command Dsolve2(p, q, r , x, c1, c2) which finds the general solution (containing two arbitrary constants c1, c2) to the second order differential equation

$$\frac{d^2y}{dx^2} + p(x)\frac{dy}{dx} + q(x)y = r(x)$$

For many of the examples in this Block both p(x) and q(x) are given constants. As an exercise use DERIVE to check the correctness of the solutions requested in the examples and guided exercises of this Block.

As a more involved exercise use DERIVE to explore the solution to

$$m\frac{d^2x}{dt^2} + k\,\frac{dx}{dt} + nx =$$

$f(t)$ with $k^2 < 4mn$ (trigonometric solutions with frequency $\omega_s = \sqrt{\frac{4mn - k^2}{2m}}$)

You will find that after an initial period the system will vibrate with the same frequency as the forcing function. The initial response is affected by the transient function (the complementary function). If there is damping in the system this will die away, due to the decaying exponential terms in the complementary function, to leave only that part of the solution arising from the particular integral. An interesting case arises when ω ≈ ωs. You will find, by plotting the solution curve, that the phenomenon of beats occurs. That is, the systen 'beats' with a frequency neither of the system frequency nor of the frequency of the forcing function. As $\omega \to \omega s$ the magnitude of the peaks and troughs of the solution curve increase.

If there is no damping in the system the response becomes unbounded as time increases in the limit $\omega = \omega s$. The system is in resonance with the forcing function. These extreme oscillations can be reduced by introducing damping into the system. Examine this phenomenon as the damping (k) is reduced. MAPLE will solve a wide range of ordinary differential equations including systems of differential equations using the command dsolve(deqns,vars,eqns) where:

deqns − ordinary differential equation in vars, or set of equations and/or initial conditions.

vars − variable or set of variables to be solved for

eqns − optional equation of the form keyword=value For example to solve

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = e^{-t} \quad y(0) = 0, \; y'(0) = 0$$

we would key in

```
> dsolve({diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=exp(-t),y(0)=0, D(y)(0)=0},y(t),type=exact);
```

MAPLE responds with

$$\frac{1 - \cos(t)}{e^t}$$

If the initial conditions are omitted MAPLE will present the solution with the correct number of arbitrary constants denoted by _C1, _C2.... Thus the general solution of

$$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 2y = e^{-t}$$

is obtained by keying in

```
> dsolve({diff(y(t),t$2)+2*diff(y(t),t)+2*y(t)=exp(-t)},y(t),type=exact);
```

and MAPLE responds with

$$y(t) = \exp(-t) + \_C1 * \exp(-t) * \cos(t) + \_C2 * \exp(-t) * \sin(t)$$

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